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29 July 1965

Office of Grants and Research Contracts
Code SC
National Aeronautics and Space Administration
Washington, D. C. 20546

Subject: Quarterly Status Report No. 5, Task Order NASr-63(07), NASA Hq. R&D 80X0108(64), MRI Project 2760-P, "Nonlinear Dynamics of Thin Shell Structures," covering the period 15 April - 14 July 1965.

Gentlemen:

Please accept this letter as a quarterly progress report for the subject contract.

In the study of large amplitude vibration and response of curved panels, there arises the problem of obtaining the solution to the nonlinear differential equation

$$y'' + \omega^2 h(y) = Q(t) \ , \ y = y(t) \quad , \qquad (1)$$

where $h(y)$ is a cubic in y and $Q(t)$ is an arbitrary forcing function (see Research Paper P-13, "Large Amplitude Vibration and Response of Curved Panels," by B. E. Cummings, Institute for Defense Analyses Research and Engineering Support Division, March 1963.) Equation (1) is seen to be a special case of the equation

$$(A_o+B_oy)y'' + (C_o+D_oy)y' - 2B_o(y')^2 + E_o + F_oy + G_oy^2 + H_oy^3 = 0 \ , \qquad (2)$$

in which the coefficients are power series in the independent variable. We developed rational approximations to the solution of (2) in the previous quarterly report. These approximations are very powerful. Observe that contrary to usual perturbation techniques, no hypotheses on the size of the parameters entering the coefficients in (2) need be made. The approximations can be used to compute the solution to any desired degree of accuracy and so are useful to evaluate poles and zeros of the solution.

In illustration, consider the simple example

$$y'' + 0.2y' + 5y + 10y^3 = 0 \ , \ y(0) = 1 \ , \ y'(0) = 0 \quad , \tag{3}$$

make the transformation $y = 1 - 7.5x^2v$ to get

$$7.5x^2v'' + (30x+1.5x^2)v' + (15+3x+262.5x^2)v$$
$$- 1687.5x^4v^2 + 4218.75x^6v^3 - 15 = 0 \ , \ v(0) = 1 \ . \tag{4}$$

Let

$$y_n = 1 - 7.5x^2v_n$$

where v_n is the n^{th} order main diagonal Padé approximation to the solution of (4). The following table exhibits the accuracy of these second order approximations.

TABLE I

x	$y(x)$ (true)	$y_2(x)$
0.00	1.0000	1.0000
0.02	0.9970	0.9970
0.04	0.9881	0.9881
0.06	0.9734	0.9734
0.08	0.9531	0.9531
0.10	0.9276	0.9276
0.12	0.8971	0.8971
0.14	0.8621	0.8621
0.16	0.8230	0.8229
0.18	0.7801	0.7801
0.20	0.7339	0.7339
0.22	0.6849	0.6849
0.24	0.6335	0.6334
0.26	0.5799	0.5798
0.28	0.5247	0.5245
0.30	0.4680	0.4677

The approximation $y_3(x)$ agrees exactly with $y(x)$ for $0 \leq x \leq 0.30$.

Note that the second order approximation is superior to a sixth order approximation developed in a previous report ("Rational Approximations to the Response of a Dynamic System Described by a Nonlinear Differential Equation," Final Report, MRI Project No. 2760-P, January 1965).

We next consider a second example which exhibits the utility of these approximations, not only as a means to compute functional values, but also as an effective procedure for obtaining information about poles of the function. Painleve's second transcendent is defined by the differential equation

$$y'' - 2y^3 - xy - u = 0 \text{ , } y(0) = 1 \text{ , } y'(0) = 0 \quad . \tag{5}$$

As is characteristic of solutions to some nonlinear differential equations, the solution to (5) has a movable pole of the first order. Approximations were obtained for the case $u = 1$. Let y_n be the n^{th} order main diagonal Padé approximation to the solution to (5). Table II compares the approximations y_n to the true solution while Table III shows the effectiveness of obtaining the poles of the function y by computing the zeros of the denominators of $y_n = \dfrac{P_n(x)}{Q_n(x)}$.

TABLE II

x	$y(x)$ (true)	$y_3(x)$	$y_5(x)$
0.0	1.0000	1.0000	1.0000
0.1	1.0152	1.0152	1.0152
0.2	1.0626	1.0626	1.0626
0.3	1.1464	1.1464	1.1464
0.4	1.2742	1.2742	1.2742
0.5	1.4592	1.4594	1.4592
0.6	1.7254	1.7265	1.7254
0.7	2.1184	2.1240	2.1184
0.8	2.737	2.762	2.737
0.9	3.834	3.953	3.832
1.0	6.311	7.080	6.289

TABLE III

($x_{n,o}$ is zero of $Q_n(x)$)

x_o (true)	$x_{3,o}$	$x_{4,o}$	$x_{5,o}$
1.16	1.14	1.17	1.16

The difficulties involved in computing the values of y and its pole are discussed in the book Introduction to Nonlinear Differential and Integral Equations by H. T. Davis, see Chapter 8.

In the next quarterly report we shall give a quantitative analysis of the solution to the very important Duffing's equation which is a special case of (2).

Very truly yours,

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(25 copies of report submitted)